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Computational Tools for Dynamic Pile Testing

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1.Introduction

Since the beginning of dynamic pile testing methods the theoretical basis has always been developed, resp. expanded according to the needs of practical application (ref.1). Whenever difficulties in the interpretation of hitherto unknown signals arose, a thorough look over the theoretical basis showed unused features of the mechanics of wave propagation and thus made the interpretation of the signals possible. This interaction of theory and practice in dynamic pile testing lead to a constant growth of the range of successful applications. The latest step in this proces has been the application to bored piles of large diameter (over 1.0 m). The first signals showed that the experience of pile driving is not sufficient for the use of dynamic testing of cast-in-situ piles.

Among the numerous theoretical investigations that have been carried out in our attempt to get a solution for bored piles two points are of general interest and will be given here :
1. General solution of a differential equation of one-dimensional wave propagation
2. Uniqueness of CAPWAP-Solutions.

The central point in theoretical investigations of dynamic pile testing is the mechanics of one dimensional wave propagation. To solve the difficulties that are connected to the application of dynamic pile testing to bored piles therefore means primarily to understand the mechanics of wave propagation. For a given mechanical model of pile and soil a differential equation can be formulated that can be solved for arbitrary initial conditions of force and velocity. The solution in form of a force-time-history or a velocity-time-history will render some knowledge of the signals to be expected in reality .

Solvability and uniqueness of solution are the most important properties of any mathematical problem. With respect to the capacity determination of piles the steps are :
1. Find a mechanical model,
2. Find a mathematical problem formulation.
The mechanical model is usually some kind of discrete spring-mass-sytem, which give a unique output signal for a given input signal. The problem is to find the best 'match' of measured and computed signals by altering the model constants. In structural dynamics this is known as a problem of systems analysis. To find the 'best match' can suitably be formulated as a problem of mathematical optimization. Well-known conditions of

solvability and uniqueness of the theory of mathematical optimization may be used to improve CAPWAP-type procedures.

2. Numerical Solution of One-Dimensional Wave Equation

The uniaxial dynamic response of a rod can be described by a partial differential equation (ref.2):

$$\ddot{u} - c^2 u'' = 0 \qquad (1)$$

with $\ddot{u} = a$: acceleration of pile element,
u'' : space derivative of strain,
$u = u(x,t)$ displacement function,
c : velocity of wave propagation.

For low frequency excitations in the linear elastic range modal analysis is used as an adequate solution method (ref.2). In the case of impact or impulsive loadings the high frequency vibrations prevail and a direct solution by d'Alemberts approach can be advantageously applied (ref.1,2).
For investigations of the movements of piles surrounded by soil the dynamic equilibrium condition (1) has to include a tangential force representing skin friction (see fig.1)

$$\mu \ddot{u} - N' + T = 0$$

with $\mu = \varrho A$: mass per unit length,
N : internal force,
T : distributed skin friction.

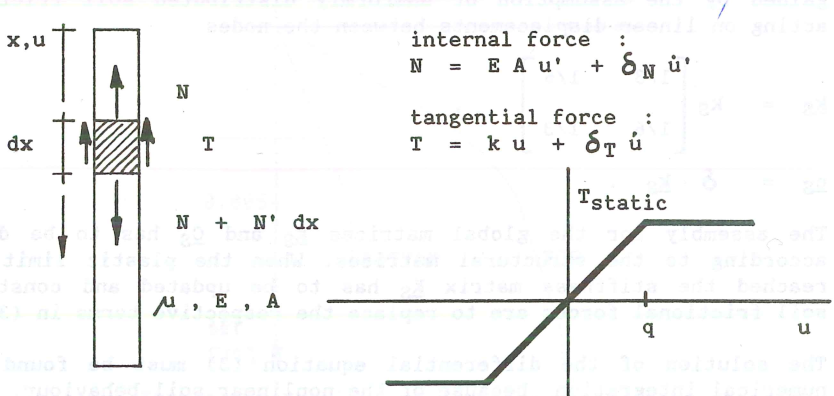


Figure 1 : Differential Pile Element

The internal force N is dependent on strains u' and in addition to (1) dependent on strain velocities $\dot{u}'$ because of material damping effects. The tangential force T represents soil friction and is taken to be dependent on the displacements u and velocities $\dot{u}$ because of soil frictional damping. The complete partial differential equation of dynamic equilibrium for the pile is therefore given as (ref.3)

$$\mu \ddot{u} - E A u'' - \delta_N \dot{u}'' + k u + \delta_T \dot{u} = 0. \qquad (2)$$

The first two terms of (2) are equivalent to (1).
Soil resistance forces are in general nonlinear and because of unloading depend on load history. The real behaviour of soil is approximated by the simple bilinear model (fig.1). For the nonlinear problem with arbitrary boundary conditions a solution can only be found by a numerical procedure (ref.2,4). A special kind of finite element formulation is adopted here.
A discretization in space of (2) can be achieved by means of the principle of virtual work (ref.2) and leads to the matrix form of the dynamic equilibrium equations which is widely used in structural dynamics

$$\underline{M}\,\ddot{\underline{u}} + (\underline{C}_S + \underline{C}_N)\,\dot{\underline{u}} + (\underline{K}_S + \underline{K}_N)\,\underline{u} = \underline{F}(t) . \qquad (3)$$

The vector $\underline{u}$ contains the nodal displacements which are only time dependend. Mass matrix $\underline{M}$ and stiffness matrix $\underline{K}_N$ are assembled by using element matrices for pin-jointed bars (ref.2,4). Assembling is done easily according to the pile geometry (ref.3). The material damping matrix $\underline{C}_N$ may be formulated mass and stiffness dependend as is usually done for Raileigh damping

$$\underline{C}_N = \alpha \underline{M} + \beta \underline{K} \qquad (4)$$

Element matrices $\underline{k}_S$ and $\underline{c}_S$ which represent soil behaviour are gained by the assumption of uniformly distributed soil friction acting on linear displacements between the nodes

$$\underline{k}_S = k_S \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} ,$$

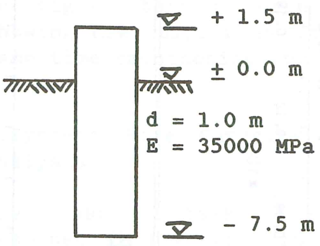
$$\underline{c}_S = \delta \cdot \underline{k}_S .$$

The assembly for the global matrices $\underline{K}_S$ and $\underline{C}_S$ has to be done according to the structural matrices. When the plastic limit is reached the stiffness matrix $\underline{K}_S$ has to be updated and constant soil frictional forces are to replace the respective terms in (3).

The solution of the differential equation (3) must be found by numerical integration because of the nonlinear soil behaviour.
Experience with the Wilson-Newmark method (ref.4) that has been tried first because of the possibility to improve the numerical stability showed good results only for small elements. For larger elements the instant change of the stiffness properties when reaching a quake induced high frequency oscillations in the acceleration that lead to false displacement and force results. Therefore an equilibrium iteration had to be implied as is used in the usual CAPWAP explicit integration scheme (ref.4). As the matrices are tridiagonal and positiv definite or semidefinite special solution procedures have been programmed.

3.Simulation of Dynamic Pile Testing

A nine meter cast-in-situ pile is used as test example (fig.2). The impact of the drop weight (m = 10 t , h = 3.0 m) has been modelled by a bell-shaped load function (ref.5).



skin friction

σ_0 = 0.5 MPa
q = 0.002 m
δ = 10^{-4} 1/s

toe resistance

σ_0 = 5.0 MPa
q = 0.005 m
δ = 10^{-3} 1/s

total static resistance : R_S = 15.7 MN

material damping : $\underline{C_N} = \underline{M} + 5.0\ 10^{-4}\ \underline{K}$

Figure 2 : Test pile

With the given pile data a static load-displacement curve can be computed. In fig.3 the load displacement relationship for top and bottom are combined and may be compared to measured curves. Of course toe resistance remains constant after the toe quake has been reached.

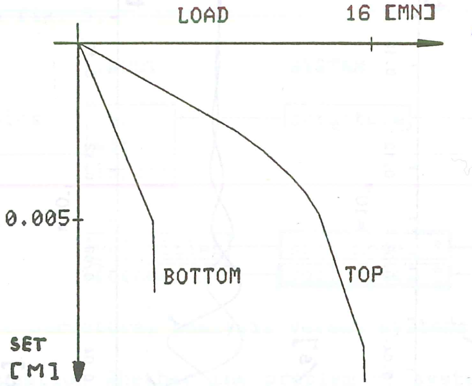


Figure 3 : Static load displacement curves

The dynamic results are given in fig.4. A 'T' is denoting values belonging to the pile top , a 'B' is denoting values belonging to the pile toe ('bottom').
In fig.4a it can be seen that the displacement of the pile top reaches a maximum of 8 mm, the pile toe 6 mm. The movement is coming to an end with a permanent set of 5 mm. Velocities (fig.4b) and accelerations (fig.4c) are converging to zero value. As can be seen by fig.4d the top force is set equal to the applied bell-shaped load function, the bottom force is limited by the

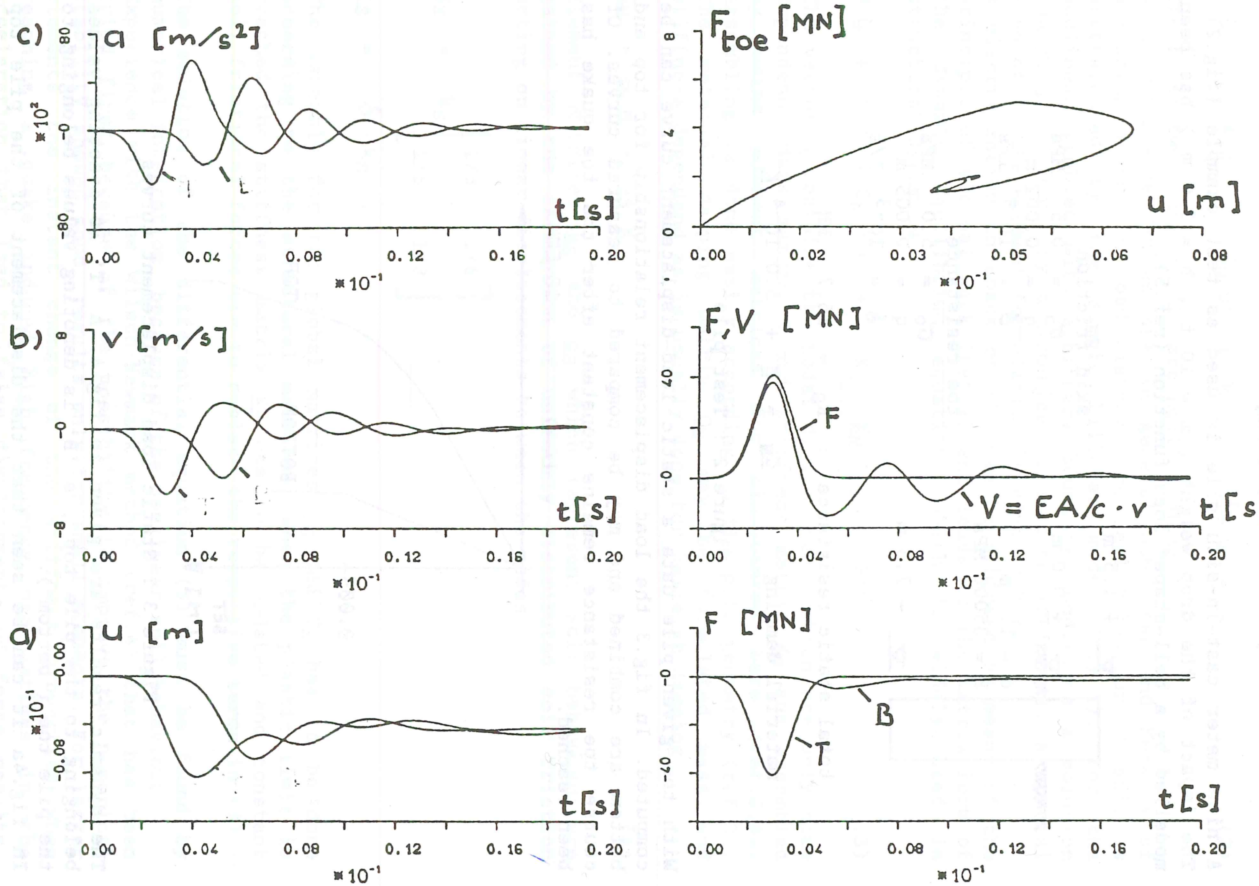


Figure 4 : Dynamic computation of pile under impact

prescribed soil condition and converges to a residual value.
Top force and proportionalized velocity are given in fig.4e. This is the basic graph of the case method, normally gained by measurements. A gradual increase of the mantle resistance as well as the upwards propagating tension wave arriving at 2L/c at the pile top can clearly be recognised.
In fig.4f bottom force versus bottom displacement is given, showing how damping is increasing the resistance force and at the same time smoothening the sharp edge of the bilinear soil model.

4.Dynamic Pile Capacity Determination as a Problem of Systems Analysis

The classical task of structural dynamics is to determine the response of a given structure to a specified exciting force. When experiments are carried out the question is mainly to determine material behaviour i.e. material constants. A redundant set of measured values describe the system behaviour, especially the transfer behaviour. In dynamic pile testing measured force and accelerations are used. The procedure is to take the one as input for a trial system, compute the output and compare it to the other measured signal. By altering the system and/or its variable properties computed and measured signals are matched. If computed and measured signals coincide within prescribed limits, the system i.e. soil model is found, the problem solved. In contrast to classical mechanics this second problem is known as Systems Analysis (ref.6). The qualitative difference of the two problems is visualized in fig. 5.

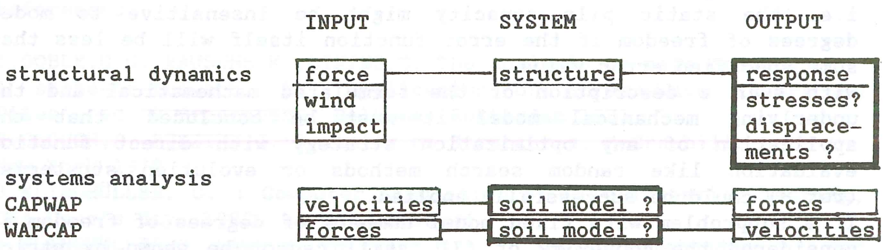


Figure 5 : Structural analysis versus systems analysis

To answer the question whether the problem of systems analysis is solvable and whether a solution will be unique a mathematical formulation of the problem has to be found. The verbal term 'coincide' has to be replaced by a suitable error function D.
Commonly used in 'best fit' problems is the sum of squared deviations (e.g. Fourier series approximations)

$$D = \int (F - F^*)^2 \, dt \qquad (5)$$

or

$$D = \sum ((F_i - F_i^*) / F_i)^2 \qquad (6)$$

The asterices * are denoting the measured or 'to be' function (F^*) resp. discrete functional values (F_i^*) of pile top force F.

Systems analysis = systems identification

Other error functions so as the sum of absolute values are possible. Experience must show which one is best.
By means of the error function D an optimization problem is formulated :

Minimize $D(\underline{Z},F,F^*)$

subject to

$$F = H(\underline{Z},v^*), \tag{7}$$

where $\underline{Z}$ is a vector containing the model constants,
H is an operator or vector-valued function, representing the computational procedure .

A problem of mathematical optimization has a solution when the domain for the variables which is described by problem (7) is bounded. The solution is unique if the domain is convex (ref.7,8). Both properties of the domain cannot easily be verified by strict mathematical proof. If the mechanical background of (7) is considered it is clearly to be recognized that the domain must be bounded i.e. there must be a finite solution for the velocities for given finite (because measured) exciting forces for a reasonable mechanical model. So it can be concluded that there is at least one solution.
If the number of degrees of freedom of the pile-soil model is included in the vector of model parameters $\underline{Z}$ the mathematical optimization problem (7) will of course not be convex. Best matches may be found for different models, a unique solution cannot be given. On the other hand , the mechanical solution , i.e. the static pile capacity might be insensitive to model degrees of freedom if the error function itself will be less than some specified error value.
With such a description of the formulated mathematical and the underlying mechanical model it must be concluded that the application of any optimization strategy with direct function evaluation like random search methods or evolution strategies (ref.8) could be successfully applied.
If a subproblem with fixed model numbers of degrees of freedom is considered the convexity of (7) still cannot be shown by strict mathematical reasoning. Convexity can only be assumed by CAPWAP experience and by the idea that the physical background of the problem may reveal some underlying energy principle that directs the solution towards the 'best match'. With the assumption of convexity strategies that use gradients can be applied (ref.7).

5. Conclusions for an improved CAPWAP procedure

A complete automatic computational procedure as has been outlined needs some time to be developped and as development and working costs cannot be estimated by now a direct approach which makes use of the theoretical considerations is suggested :

1. Choose a start model and a start time range
2. Choose an error function and install procedures to compute its value
3. Carry out CAPWAP computations with respect to a single parameter (e.g. total static resistance, parameter for toe to skin resistance etc.)
4. Compute the error value and numerical gradients with respect to the chosen parameter,
5. Determine a step towards the 'best match', i.e. the minimum error value, using the gradients
6. If the minimum is found, choose another parameter (e.g. damping) and proceed with 3.
7. If parameter variations indicate that a minimum has been found for the model, an enlarged time range might be chosen for check of convergence,
8. For check of sensitivity of the static resistance try another model

A first step is of course to compute numerical error values and decide model alteration on this basis. As the author got to know recently this is already tried by Pile Dynamics Int.. The next would be to compute numerical gradients and respective increments of variables.
With the implementation of the additional computations not only a faster way of getting CAPWAP-matches could result but also a more reliable capacity prediction can be achieved because of the use of an objective numerical value to define the 'best match'.

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